History of the Vacuum Tube Theory, SPICE models

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Introduction

Theory follows experiment. Many people believe that Lee De Forest could not explain the function of his thermionic audion tube. But the theory did catch on. Heinrich Barkhausen wrote that the "discovery" of the pentode tube was guided by theoretical understanding. A lot of the theoretical and practical work for triode and tetrode was done while the first world war made publication "in time" impossible. The french TM tube "triode militaire" and the U.S.A. tubes VT-1 to VT-5 were developed and produced in the war time for the military. These tubes were among the first high vacuum tubes. This paper uses only german and U.S.A. documents. The author knows that scientists in French and in the U.K. worked at vacuum tubes, too.

Child

In 1911 C. D. Child published "Discharge from hot CaO". He plotted the I-V characteristic of a gaseous or "low" vacuum diode with an oxide cathode.

$$I = \frac{1}{9\cdot\pi}\cdot\sqrt{\frac{2\cdot\varepsilon}{m}}\cdot\frac{V_1^{\frac{3}{2}}}{x_1^{2}} \qquad\qquad I_e = G\cdot V_d^{\frac{3}{2}}$$

Left is the Child law from the publication, right is an alternative formulation. The emission (cathode) current I_e depends on the perveance G and the diode voltage V_d between anode (plate) and cathode.

Barkhausen

Heinrich Barkhausen was very influencial in the german speaking countries. His "Elektronenröhre" books were continously published and updated from 1923 to 1965. In 1919 he published a pre-print of his book in the "Jahrbuch der drahtlosen Telegraphie und Telefonie" under the title "Die Vakuumröhre und ihre technischen Anwendungen". A foot note explains that this paper was part of a report he wrote for the "Inspektion des Torpedowesens" in summer 1917. He begins with three equations from Richardson, Langmuir and Schottky that describe the diode I-V characteristic.

Saturation current: $I_S = A\cdot F\cdot\sqrt{T}\cdot e^{\frac{-B}{T}}$ valid for $E > E_S$

Space charge current: $I = C\cdot E^{\frac{3}{2}}$ valid for $E_S > E > 0$

"Anlauf" current: $E_0 = 8.6\cdot 10^{-5}\cdot T$ $I = I_0\cdot e^{\frac{E}{E_0}}$ valid for $E < 0$

The space charge current is the Child law. The saturation current states that there is an upper limit for the diode current. The "Anlauf" current explains that there is always conduction between anode cathode as long as the cathode is heated. But the Anlauf current is very small. The Anlauf current is important for a tube volt meter, but can be neglected for a power output tube.
Barkhausen sees a difference between cathode (emission) current J_e and the anode current J_a. The Barkhausen "Steuer" voltage E_{st} has the same influence on the cathode current as the diode voltage in the Child law. The "Durchgriff" D is the reciprocal of the amplification constant μ.

$$J_e = J_g + J_a \qquad I_e = I_g + I_a$$

$$E_{st} = E_g + D \cdot E_a \qquad V_d = V_g + \frac{V_a}{\mu}$$

$$J_e = 0.015 \cdot \frac{1}{r} \cdot E_{st}^{\frac{3}{2}} = 0.015 \cdot \frac{1}{r} \cdot \left(E_g + D \cdot E_a\right)^{\frac{3}{2}} \qquad I_e = G \cdot \left(V_g + \frac{V_a}{\mu}\right)^{\frac{3}{2}}$$

Barkhausen did investigate the influce of the screen grid in a tetrode on the emission current.

$$J_e = C \cdot E_{st}^{\frac{3}{2}} = C \cdot \left[E_{g1} + D_g \cdot \left(E_{g2} + D_a \cdot E_a\right)\right]^{\frac{3}{2}} \qquad I_e = G \cdot \left(V_g + \frac{V_{g2} + \frac{V_a}{\mu_a}}{\mu_g}\right)^{\frac{3}{2}}$$

van der Bijl

In U.S.A. Hendrik Johannes van der Bijl worked at the vacuum tube theory. He published in 1920 the book "The thermionic vacuum tube and its application". We must assume that the world war did prevent an earlier publication date. Van der Bijl did add a term for the contact electromotive force (contact potential difference) and for the stray field to the Child law. Both influences he lumped into constant ε.

$$E_s = \frac{E_p}{\mu} + E_g + \varepsilon \qquad V_d = \frac{V_a}{\mu} + V_g + V_\varepsilon$$

Van der Bijl started perhaps the confusion about emission current and plate current by using I_p instead of I_e in his triode equation. That van der Bijl used β instead of 3/2 was only to introduce μ. Later it was found that the Child law exponent can range between 1.2 and 2.5.

$$I_p = \alpha \cdot \left(\frac{E_p}{\mu} + E_g + \varepsilon\right)^{\beta} \qquad I_e = G \cdot \left(\frac{V_a}{\mu} + V_g + V_\varepsilon\right)^{m}$$

The amplification constant μ is defined as the variation in plate voltage V_a and grid voltage V_g that will produce the same emission current.

$$\frac{dE_p}{dE_g} = -\mu \qquad \mu\left(I_e = const\right) = \frac{V_{a_1} - V_{a_0}}{V_{g_0} - V_{g_1}}$$

Van der Bijl described a method to estimate ε. He wrote "From the value of the plate voltage for which the current is just reduced to zero we get":

$$\varepsilon = -\left(\frac{E_p}{\mu} + E_g\right) \qquad V_\varepsilon\left(I_e = 0\right) = \frac{-V_a}{\mu} - V_g$$

Tank

In 1922 Franz Tank from Zürich, Switzerland, did publish "Zur Kenntnis der Vorgänge in Elektronenröhren" in the "Jahrbuch der drahtlosen Telegraphie und Telephonie". His topic was the distribution of cathode current J_s between anode current J_a and grid current J_g.

$$J_a + J_g = J_s$$

$$\frac{J_g}{J_a} = \Phi\left(\frac{E_g}{E_a}\right) \qquad J_a = J_s \cdot \frac{1}{1 + \phi\left(\frac{E_g}{E_a}\right)} \qquad J_g = J_s \cdot \frac{\phi\left(\frac{E_g}{E_a}\right)}{1 + \phi\left(\frac{E_g}{E_a}\right)} \qquad \phi = \mu \cdot \sqrt{\frac{E_g}{E_a}}$$

The function Φ() is estimated by μ times the square root of the voltages. The Tank μ is different from the van der Bijl μ. Both are multiplication constants. But the first is called "Bedeckungsfaktor" by Barkhausen and the other is the amplification constant. The Tank equations are invalid for V_a=0. Tank presented another form of the equations that is only invald if V_a and V_g are both zero. The empirical found square root function is replaced by a power function to give flexibility.

$$J_a = J_s \cdot \frac{\sqrt{E_a}}{\sqrt{E_a} + \mu \cdot \sqrt{E_g}} \qquad I_a = I_e \cdot \frac{V_a^{\ n}}{V_a^{\ n} + B \cdot V_g^{\ n}} = G \cdot \left(\frac{V_a}{\mu} + V_g + V_\varepsilon\right)^m \cdot \frac{V_a^{\ n}}{V_a^{\ n} + B \cdot V_g^{\ n}}$$

$$J_g = J_s \cdot \frac{\mu \cdot \sqrt{E_g}}{\sqrt{E_a} + \mu \cdot \sqrt{E_g}} \qquad I_g = I_e \cdot \frac{B \cdot V_g^{\ n}}{V_a^{\ n} + B \cdot V_g^{\ n}} = G \cdot \left(\frac{V_a}{\mu} + V_g + V_\varepsilon\right)^m \cdot \frac{B \cdot V_g^{\ n}}{V_a^{\ n} + B \cdot V_g^{\ n}}$$

SPICE Tetrode/Pentode model

The Tank equations for current distribution are not only valid for triode tubes but for tetrode and pentode tubes. The Tank equations can be used without modifications if
- the grid current I_g is ignored for the current distribution,
- the suppressor grid I_{g3} is hard wired to the cathode and can be neglected,
- the influence of anode voltage V_a on emission current I_e is neglected.

$$I_a = G \cdot \left(\frac{V_{g2}}{\mu} + V_g + V_\varepsilon \right)^m \cdot \frac{V_a^{\,n}}{V_a^{\,n} + B \cdot V_{g2}^{\,n}}$$

$$I_{g2} = G \cdot \left(\frac{V_{g2}}{\mu} + V_g + V_\varepsilon \right)^m \cdot \frac{B \cdot V_{g2}^{\,n}}{V_a^{\,n} + B \cdot V_{g2}^{\,n}}$$

A model generator has to estimate perveance G, amplicication constant μ, contact potential V_ε, Child law exponent m, current distribution factor b and current distribution exponent n.

Pentode EL84/6BQ5 model

Since 1953 the european tube EL84 was the output pentode in most radio receivers. This tube was available in U.S.A. as 6BQ5 and in Russia as 6p14p. The typical anode and screen grid voltage was 250 volts. The class A anode current anode was 36mA for a grid voltage of -8.4V. From the triode characteristic we fetch the grid voltage and anode voltage values for an anode current of 35mA.

Some prefixe: $m := 10^{-3}$

$I_{a\mu} = 35m \qquad V_{g\mu} := (0 \quad -2.5 \quad -5 \quad -7.5 \quad -10)^T$

$V_{a\mu} := (86 \quad 131 \quad 175 \quad 223 \quad 268)^T$

The amplification constant μ is estimated first. The datasheet value μ_{g2g1} is 19.

$$i := 0 .. \text{letzte}(V_{a\mu}) - 1 \qquad \mu'_i := \frac{V_{a\mu_i} - V_{a\mu_{i+1}}}{V_{g\mu_{i+1}} - V_{g\mu_i}}$$

$\mu'^T = (18 \quad 17.6 \quad 19.2 \quad 18)$

$\mu := \text{mittelwert}(\mu') \qquad \mu = 18.2$

The contact voltage V_ε is measured at an anode current of zero.

$$I_{e\varepsilon} = 0 \qquad V_{g\varepsilon} := (-2.5 \quad -5 \quad -7.5 \quad -10 \quad -12.5 \quad -15)^T$$

$$V_{a\varepsilon} := (30 \quad 80 \quad 130 \quad 163 \quad 206 \quad 224)^T$$

$$V_{\varepsilon'} := \overrightarrow{\left(-V_{g\varepsilon} - \frac{V_{a\varepsilon}}{\mu}\right)}$$

$$V_{\varepsilon'}^T = (0.85 \quad 0.6 \quad 0.36 \quad 1.04 \quad 1.18 \quad 2.69)$$

$$V_\varepsilon := \text{mittelwert}\left(V_{\varepsilon'}\right) \qquad V_\varepsilon = 1.12$$

The perveance G and the Child law exponent m are estimated with a straight line fit of the I-V characteristic in a log-log coordinate system.

$$y = a \cdot x^b \qquad \ln(y) = \ln\left(a \cdot x^b\right) = \ln(a) + B \cdot \ln(x)$$

The I-V curve for the parameter Vg=-7.5 from the triode characteristic was used.

$$V_{gm} := -7.5 \qquad I_{am} := (5m \quad 10m \quad 20m \quad 30m \quad 50m)^T$$

$$V_{am} := (155 \quad 172 \quad 196 \quad 214 \quad 245)^T$$

The anode and the screen grid are connected for a pentode in triode configuration. The suppressor grid is connected to the cathode. The grid current can be neglected at a grid voltage of -7.5V. The triode anode current is therefore the emission current.

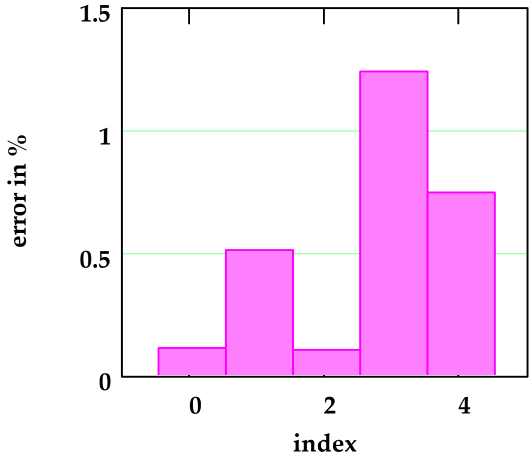
$$I_e = G \cdot \left(\frac{V_{am}}{\mu} + V_{gm} + V_\varepsilon\right)^m \qquad \ln\left(I_e\right) = m \cdot \ln\left(\frac{V_{am}}{\mu} + V_{gm} + V_\varepsilon\right) + \ln(G)$$

$$\begin{pmatrix} \ln G \\ m \end{pmatrix} := \text{linie}\left(\ln\left(\frac{V_{am}}{\mu} + V_{gm} + V_\varepsilon\right), \ln\left(I_{am}\right)\right) \qquad G := e^{\ln G}$$

$$G = 1.15 \times 10^{-3} \qquad m = 1.93$$

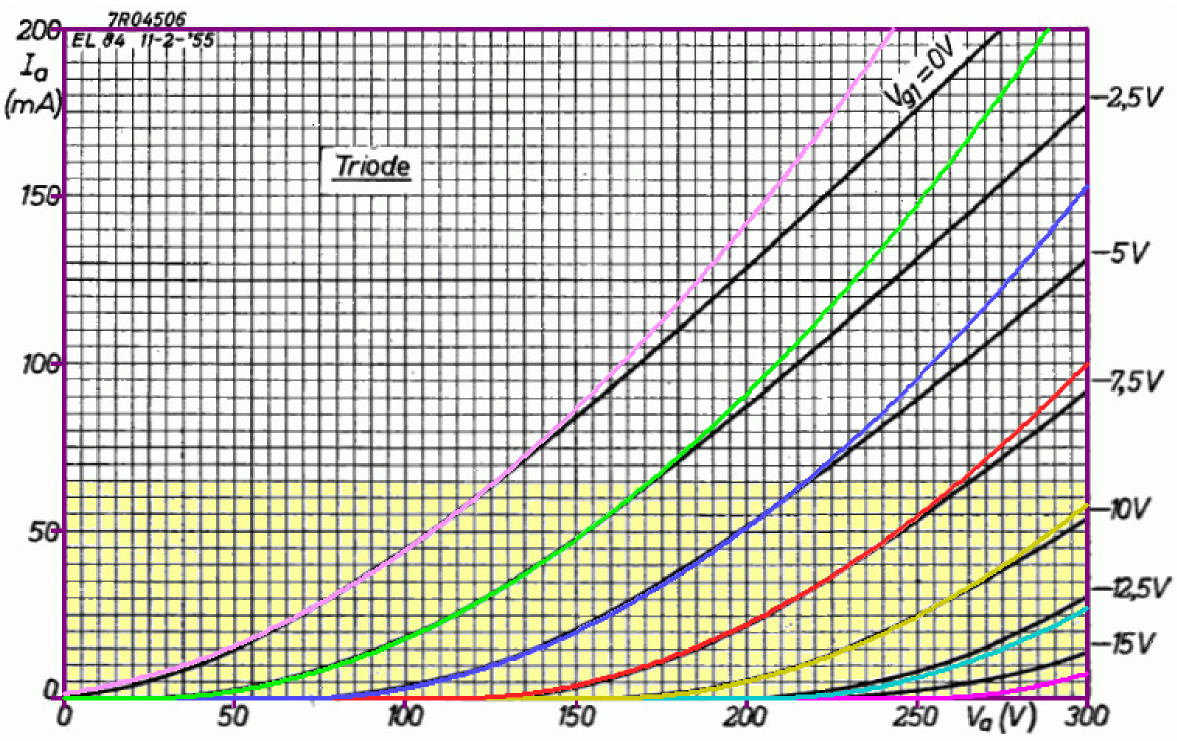
The model error is small. But we have to compare the model with the I-V characteristic.

$\mathbf{i} := 0 .. \mathbf{letzte}(\mathbf{I_{am}})$

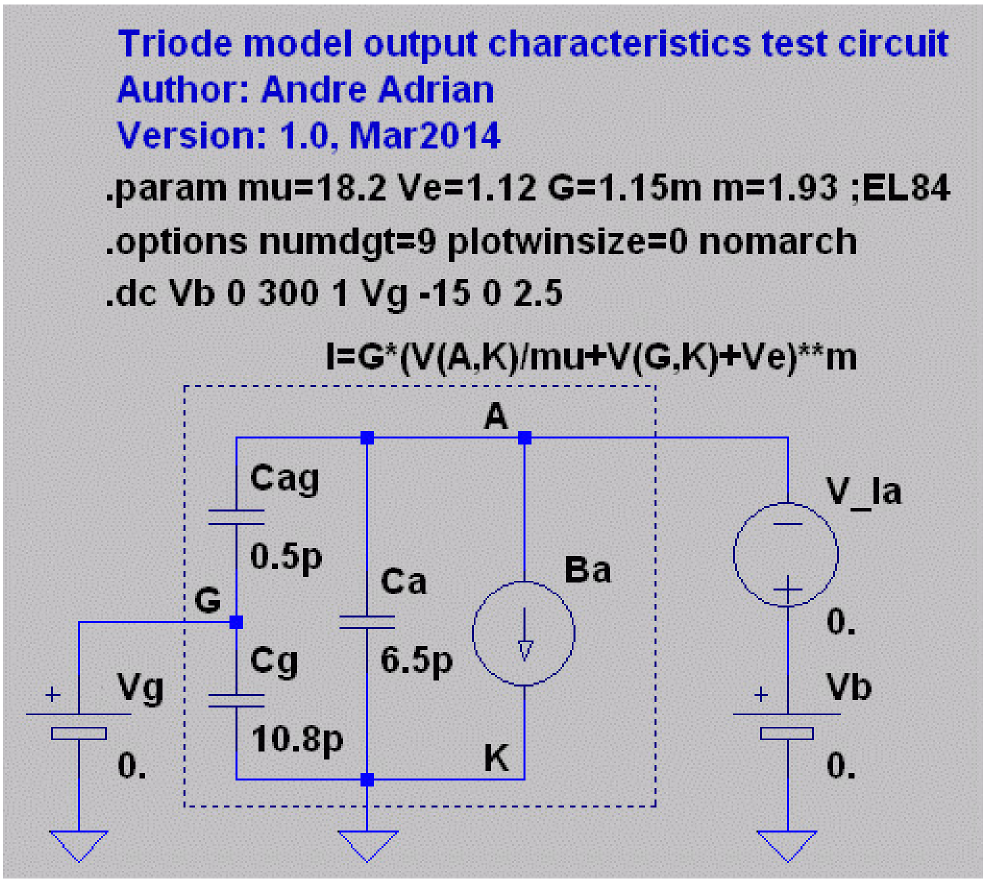


$\mu = 18.2 \qquad V_\varepsilon = 1.12 \qquad G = 1.15 \times 10^{-3} \qquad m = 1.93$

$$I_e = G \cdot \left(\frac{V_a}{\mu} + V_g + V_\varepsilon \right)^m$$



Picture 1: EL84 triode characteristic with SPICE model overlay. The yellow area is limited by the maximum emission current of 65mA.



Picture 2: LTSpice simulation of EL84 in triode mode with output test circuit.

The pentode I-V characteristic is used to estimate B and n.

$$V_{gn} := -8 \qquad I_{an} := (20m \quad 25m \quad 30m \quad 35m \quad 40m)^T \qquad V_{g2} := 250$$

$$V_{an} := (5 \quad 25 \quad 41 \quad 60 \quad 190)^T$$

The tetrode equation for I_a with only a single n is brought into the form $a \cdot x^b$ for a straight line fit.

$$I_a = \frac{G \cdot \left(\frac{V_{g2}}{\mu} + V_g + V_\varepsilon\right)^m}{1 + B \cdot \left(\frac{V_{g2}}{V_a}\right)^n} \qquad \frac{1}{I_a} = \frac{1 + B \cdot \left(\frac{V_{g2}}{V_a}\right)^n}{G \cdot \left(\frac{V_{g2}}{\mu} + V_g + V_\varepsilon\right)^m}$$

$$\frac{G\cdot\left(\frac{V_{g2}}{\mu}+V_g+V_\varepsilon\right)^m}{I_a}-1 = B\cdot\left(\frac{V_{g2}}{V_a}\right)^n$$

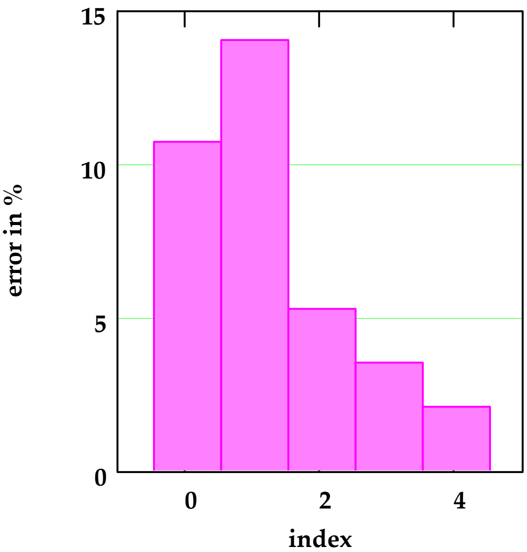
$$\ln\left[\frac{G\cdot\left(\frac{V_{g2}}{\mu}+V_g+V_\varepsilon\right)^m}{I_a}-1\right] = n\cdot\ln\left(\frac{V_{g2}}{V_a}\right)+\ln(B)$$

$$\begin{pmatrix}\ln B\\ n\end{pmatrix} := \text{linie}\left[\ln\left(\frac{V_{g2}}{V_{an}}\right),\ln\left[\frac{G\cdot\left(\frac{V_{g2}}{\mu}+V_{gn}+V_\varepsilon\right)^m}{I_{an}}-1\right]\right] \qquad B := e^{\ln B}$$

$B = 0.18$ $\qquad$ $n = 0.57$

The model error is acceptable. But we have to compare the model with the I-V characteristic.

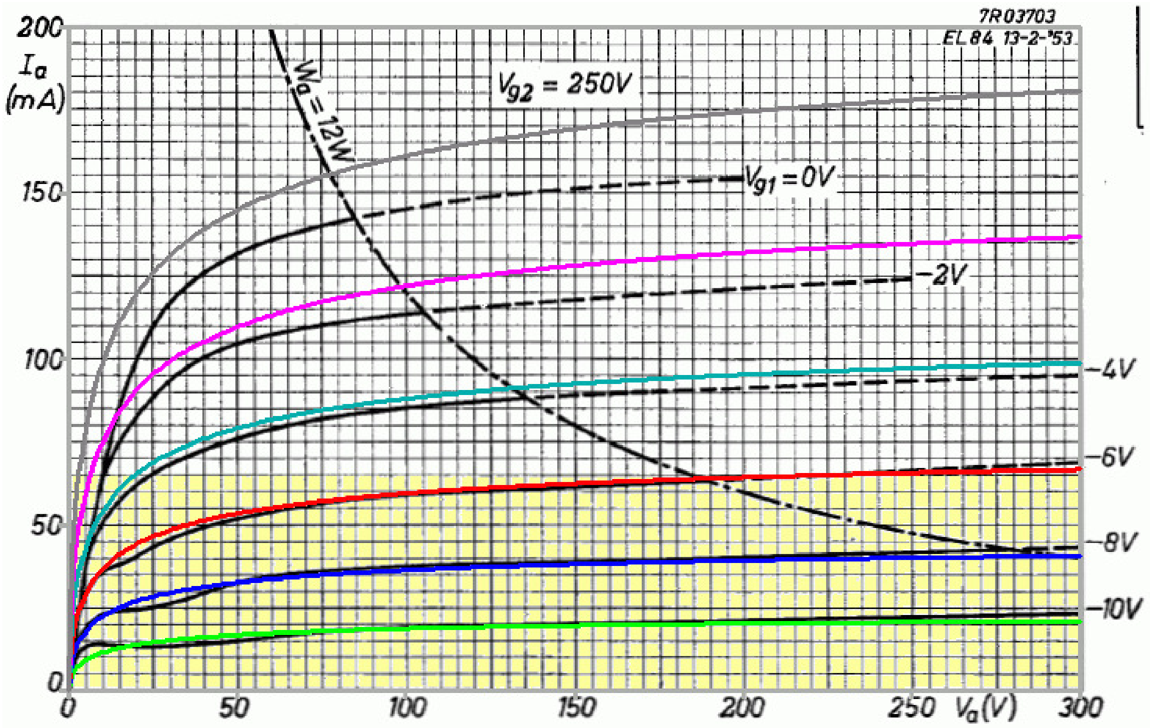
$i := 0 .. \text{letzte}(I_{an})$



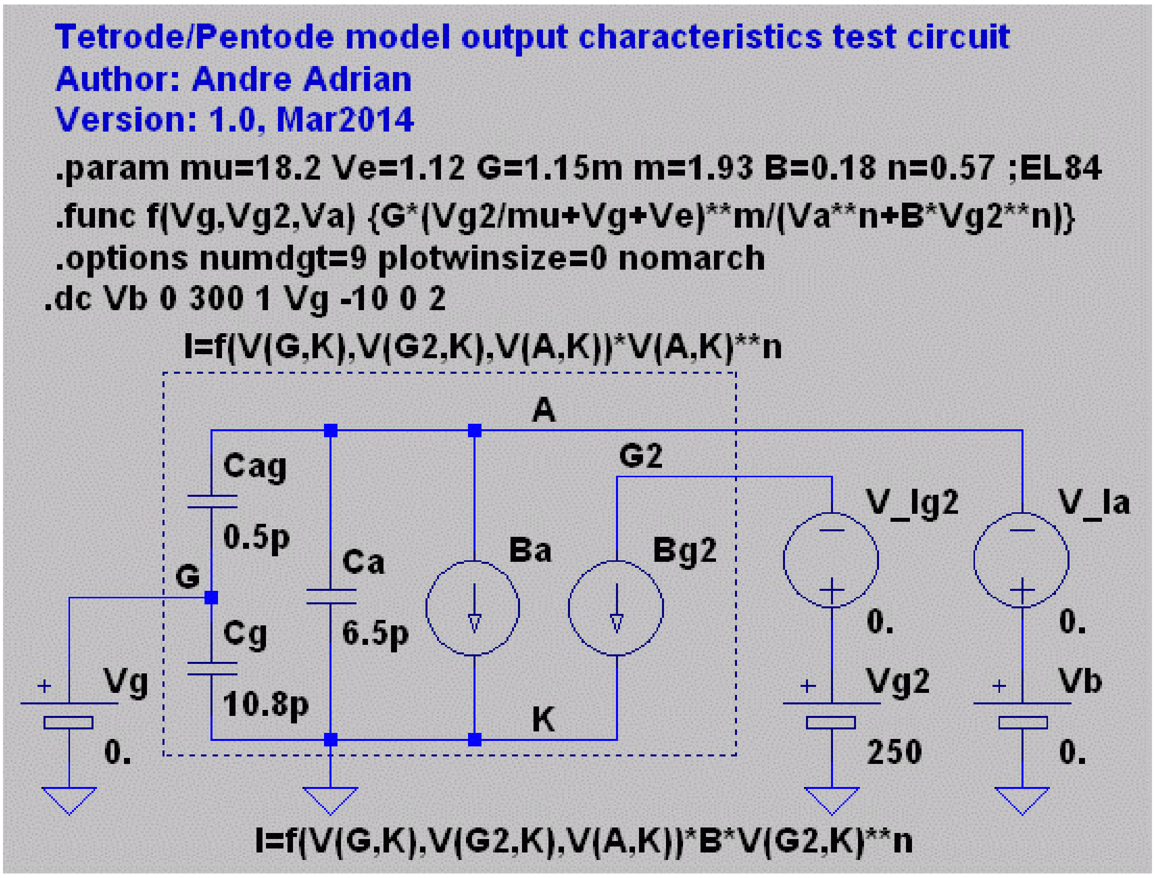
The function f() contains the common part of the I_a and I_{g2} equations. The only purpose is convienience.

$$f(V_g, V_{g2}, V_a) := \frac{G\cdot\left(\frac{V_{g2}}{\mu}+V_g+V_\varepsilon\right)^m}{V_a^{\,n}+B\cdot V_{g2}^{\,n}}$$

$$I_a(V_g, V_{g2}, V_a) := f(V_g, V_{g2}, V_a)\cdot V_a^{\,n} \qquad I_{g2}(V_g, V_{g2}, V_a) := f(V_g, V_{g2}, V_a)\cdot B\cdot V_{g2}^{\,n}$$



Picture 3: EL84 pentode characteristic with SPICE model overlay. The yellow area is limited by the maximum emission current of 65mA.



Conclusion

The vacuum tube theory was developed between 1911 (Child law) and 1922 (Tank current distribution). At this time tetrode tubes were rare and pentode tubes were not invented. The old equations work well if combined with a straight line fit. The straight line fit uses the fact that a power law function is a straight line in a log-log coordinate system. The model is no "black-box" model that just tries to match some arbitrary curves. It is based on the "inner working" of a vacuum tube. That electrons are modelled as "ballistic cannon balls" is good enough for our purposes.